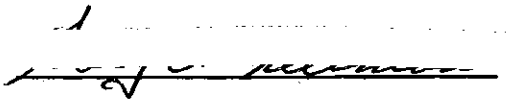
In presenting the dissertation as a partial fulfillment of the requirements for an advanced degree from the Georgia Institute of Technology, I agree that the Library of the Institute shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to copy from, or to publish from, this dissertation may be granted by the professor under whose direction it was written, or, in his absence, by the Dean of the Graduate Division when such copying or publication is solely for scholarly purposes and does not involve potential financial gain. It is understood that any copying from, or publication of, this dissertation which involves potential financial gain will not be allowed without written permission.



7/25/68

DRAG OF SPHERES IN AQUEOUS SOLUTIONS OF THE LONG-CHAIN POLYMER "POLYHALL 295"

A THESIS

Presented to

The Faculty of the Graduate Division

by

Gary Irving Kellman

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Civil Engineering

Georgia Institute of Technology

June, 1969

DRAG OF SPHERES IN AQUEOUS SOLUTIONS OF THE LONG-CHAIN POLYMER "POLYHALL 295"

Approved:

Chairman

Date approved by Chairman: 5/30/69

ACKNOWLEDGMENTS

The author's gratitude is extended to all who helped make this milestone in his life, the attainment of a Masters' degree, possible. Special appreciation is extended to Julius and Ruth Kellman, the parents of the author, and Dr. Ian A. Kellman and Mr. Robert M. Kellman, the brothers of the author, as they have always been, and always will be, a source of inspiration and pride in the author's life.

Appreciation is also extended to Dr. Paul G. Mayer, Dr. Arthur L. Bennett, and Dr. H. Clay Lewis for their interest, encouragement, and guidance, in the preparation of this thesis. Dr. Mayer is also thanked for introducing the author to this very interesting new area of research.

The School of Civil Engineering is thanked for its support of the author and of his research, in the award of a National Science Foundation Traineeship.

Gratitude is extended to Joan Mauro and Brenda Burbank for their incalculable aid in the preparation of this thesis.

Sincere warmth is extended to Ben and Jackie Hirsch and their four wonderful children, Shoshana, Adina, Michal, and Raphael, and to the families of Mr. & Mrs. David Kaplan, Mr. & Mrs. Sidney Kaplan, Rev. & Mrs. Isaac Goodfriend, Mr. & Mrs. William Gris, and Mel Eskowitz, for extending to the author the warmth and delight of their homes during his stay in Atlanta.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

CHAPTER I

INTRODUCTION

The unusual hydrodynamic properties exhibited by solutions of long-chain macromolecules in water have stimulated much interest in many engineers and scientists in recent years. The engineering importance of these long-chain polymers is founded upon the fact that the addition of very small amounts of these polymers to solvents, such as water, can produce drastic decreases in the fluid dynamic drag.

The flow condition for which these reductions have been most spectacular, and for which the most extensive experimental and theoretical work has been performed, is that of turbulent pipe flow. This flow condition is such that the drag is primarily due to turbulent skin friction. Although much data have been compiled for this type of flow, there is still no clear understanding, at this time, of the physical mechanisms responsible for the observed effects.

Much less attention has been paid by researchers to other types of flow, such as the flow around spheres. Research with this type of flow might very well add new insights into the hydrodynamic mechanisms responsible for the reduction of drag in turbulent pipe flow with the addition of long-chain polymers.

The few experiments which have been performed on flow about spheres used the long-chain polymer additives poly(ethylene oxide) and guar gum.

One commercially available long-chain polymer is "Polyhall 295." This additive has been reported by the manufacturer, Stein, Hall & Co., of New York City, to reduce drag in pipelines. Some experimentation with this polymer was carried out by J. B. Jackson (1) in the Hydraulics Laboratory of the Civil Engineering Department, The Georgia Institute of Technology, in March, 1967. Jackson's experiments studied the effects of the addition of Polyhall 295 to water flowing in pipes and to water flowing through porous media.

The purpose of this thesis was to formulate a series of experiments to study how the long-chain polymer Polyhall 295 would affect the flow about spheres. These studies, it was hoped, would help shed some light on the mechanisms involved in the drag reduction caused by long-chain polymer addition.

CHAPTER II

LITERATURE REVIEW

Experiments in Turbulent Flow

Before proceeding into the discussion of the experiments with spheres, it will be helpful to review some of the more important features of the turbulent flow of polymer solutions.

For fully developed turbulent pipe flow, a relationship for the friction factor, f, has been developed by White (2) based on the Prandtl two zone model. This model represents turbulent pipe flow as two regions; first, a turbulent core region where inertia forces dominate, and second, a thin sublayer adjacent to the wall where viscous forces dominate (17). The formula given by White is:

$$\frac{1}{\sqrt{f}} = \frac{1}{k\sqrt{2}} [\ln.Re\sqrt{f} - \ln.2\sqrt{2} + kRo - \ln.Ro - 3/2] \qquad (1)$$

in which f is the friction factor, Re is the pipe flow Reynolds number and is equal to Vd/ν, Ro is the sublayer Reynolds number, a parameter which represents the stability of the laminar sublayer and is expressed as:

$$Ro = \frac{z_o u^*}{\nu} \qquad (2)$$

and k is the mixing constant, a parameter which is determined by the equation:

$$\frac{u}{u^*} = \frac{1}{k} \ln. \frac{zu^*}{\nu} + \text{Constant} \qquad (3)$$

and is representative of the velocity profile within the turbulent core region. The other parameters are:

d = Pipe diameter

u = Axial velocity

V = Mean axial velocity

u^* = Friction velocity

z = Distance from wall

z_o = Thickness of boundary layer

ν = Kinematic viscosity

From Equation (1) it is apparent that for a reduction in f to occur, k must decrease, Ro must increase or both may change.

Both Elata et al. (3) and Ernst (4), took velocity profile measurements of polymer solutions flowing in various sized circular pipes.

Elata's experiments were performed using guar gum solutions of between 50 and 10,000 wppm in four pipe sizes ranging from 12.18mm (.479 inch) to 50.70mm (1.99 inch) in diameter. Ernst's experiments were performed using a 500 wppm concentration of carboxymethyl-cellulose (CMC) in a 0.650 inch and a 1.417 inch pipe.

Both experimenters reported a decrease in pipe friction with the

addition of polymer, no noticeable change in the shapes of the velocity profiles for polymer solutions from those of Newtonian fluids, and decreasing friction factor with decreasing pipe diameter. Elata also reported a decrease in drag with increasing polymer concentration.

As the shape of the velocity profiles did not change, the value of k from Equation (3) must have remained constant. Thus, since a decrease in the value of f was reported, Equation (1) indicates that the value of Ro must have increased.

Elata et al. suggested that the increase in the value of Ro and thus the increased stability and thickness of the laminar sublayer, was caused by the damping of high frequency disturbances within the sublayer. They theorized that at very high shear rates, such as those existing in the laminar sublayer, the normally "randomly-coiled" polymer molecules became extended, and local velocity fluctuations were dampened out by the polymer molecules.

That increased stability does, in fact, exist in highly sheared polymer solutions may be concluded from the experiments of Gadd (13), who showed that the transition to turbulence in a submerged laminar jet ejected from a capillary was delayed when small amounts of polymers were added to the fluid.

White (2), in an effort to confirm the conclusions of Elata et al. (3) and Ernst (4), experimented with solutions of guar gum and poly-(ethylene oxide) in artificially roughened pipes. The guar gum, which was found by Elata et al. not to change the value of k even for a 10,000 wppm solution, did not have any effect on the pressure drop along the artificially roughened pipe. The polyox, (poly(ethylene oxide)),

however, did cause drag reductions in the pipe with the roughened walls. Furthermore, the drag reduction only occurred above a threshold Reynolds number. This threshold Reynolds number was believed to be the minimum Reynolds number for which the shear is sufficient to elongate the random-coiling polymer molecules.

To confirm this last theory, a series of pipe flow tests were performed (2) using a linear polymer, that is, a non-coiling polymer which is normally elongated in solution without the application of a shear stress. As expected these polymer solutions did not exhibit a threshold Reynolds number.

Hoyt and Fabula (5) devised an experiment to measure the change in skin friction drag with the addition of long-chain polymers to solution, they measured the torque necessary to drive a disk immersed in the solution. For a given disk diameter and angular velocity, the torque required to move the disk is a measure of the skin friction drag on the disk. They found by experimenting with many different polymers, that the most successful drag-reducing additives were those with long molecules of high molecular weight with few side branches and good solubility. They also found that the maximum drag was obtained with extremely low concentrations (less than 100 wppm), and that as the concentration was raised above that needed for optimum effect, the drag reduction decreased slowly with increasing concentration.

Figure 1 illustrates the percentage drag reduction for flow in pipes as a function of the polymer concentration. The data are the results of tests performed by Hoyt and Fabula (5) for Polyox WSR 301 (a commercially available form of poly(ethylene oxide)) solutions flowing

in a 0.11 cm pipe with a flow velocity of 12.6 m/sec., and of the Stein, Hall & Co., (18) for solutions of Polyhall 295 flowing in a 0.125 inch diameter pipe with a flow rate of 10-12 gph.

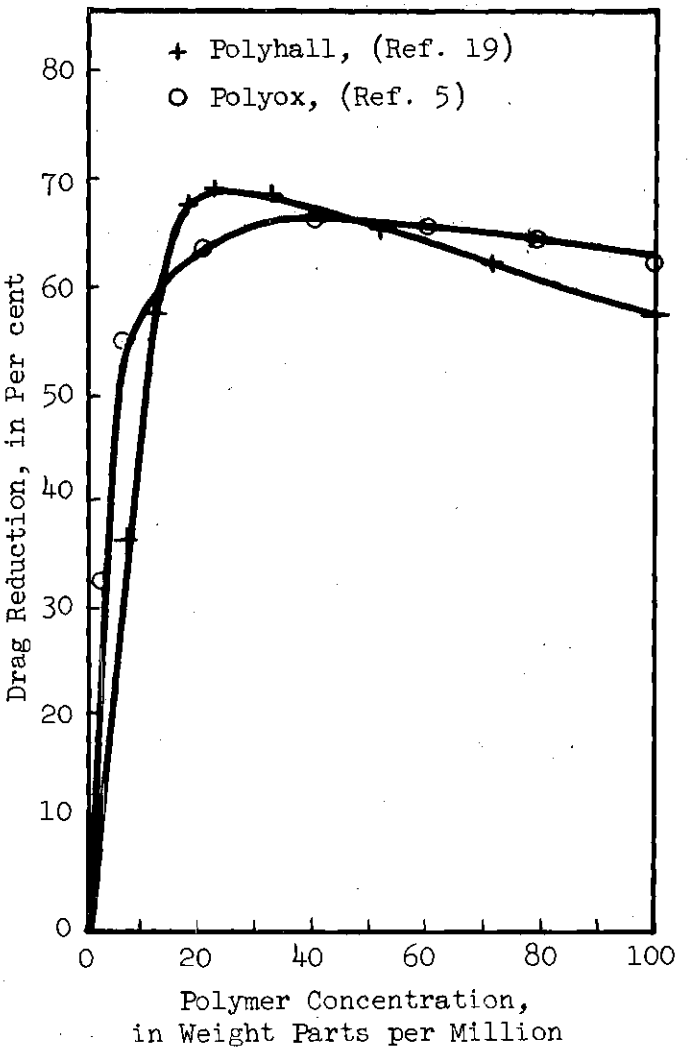


Figure 1. Drag Reduction for Polymer Solution Flow in Pipes

Georgia Tech Experiments with Polyhall 295

J. B. Jackson (1) conducted a series of three experiments with Polyhall 295. The first of these was designed to study the effect of the polymer in a laminar flow regime. In order to obtain purely viscous

flow, the solution was allowed to flow through a porous medium. He found in this case that the discharge decreased with increasing concentration. One explanation of this phenomenon is that the polymer molecules bridge and block the voids in the porous medium.

The second and third experiments were in turbulent pipe flow. In these experiments two different centrifugal pumps were employed in a recirculating system to pass the polymer through a 1.61 inch ID pipe. For example, Jackson reported a 68 per cent reduction in the friction factor for concentrations above 300 wppm when the larger pump and a flow rate of 0.167 cfs was employed. When the smaller pump was used at a flow rate of 0.0585 cfs, the reduction of the friction factor was 24 per cent for the same polymer concentration (See Figure 2).

Jackson noted that immediately upon the addition of polymer, there was a noticeable higher reduction in friction factor. This condition lasted only a short time, and Jackson reasoned that this was probably due to the physical degradation of the polymer as it repeatedly passed through the pump. Jackson also attempted some velocity profile measurements, but was unable to get any meaningful results. This failure seemed to have been due to polymer clogging the small opening of the traversing pitot tube.

Stein, Hall & Co., Inc., the manufacturer of Polyhall, has also reported time-varying pressure drop reductions for Polyhall 295 (See Figure 3). These experiments were performed in a recirculation system similar to that of Jackson, using a 0.269 inch ID pipe, at a temperature of 70°F. The maximum pressure drop reduction recorded was for 250 wppm of Polyhall 295 in a solution of A.P.I. Brine (8%NaCl and 2.5%$CaCl_2$)

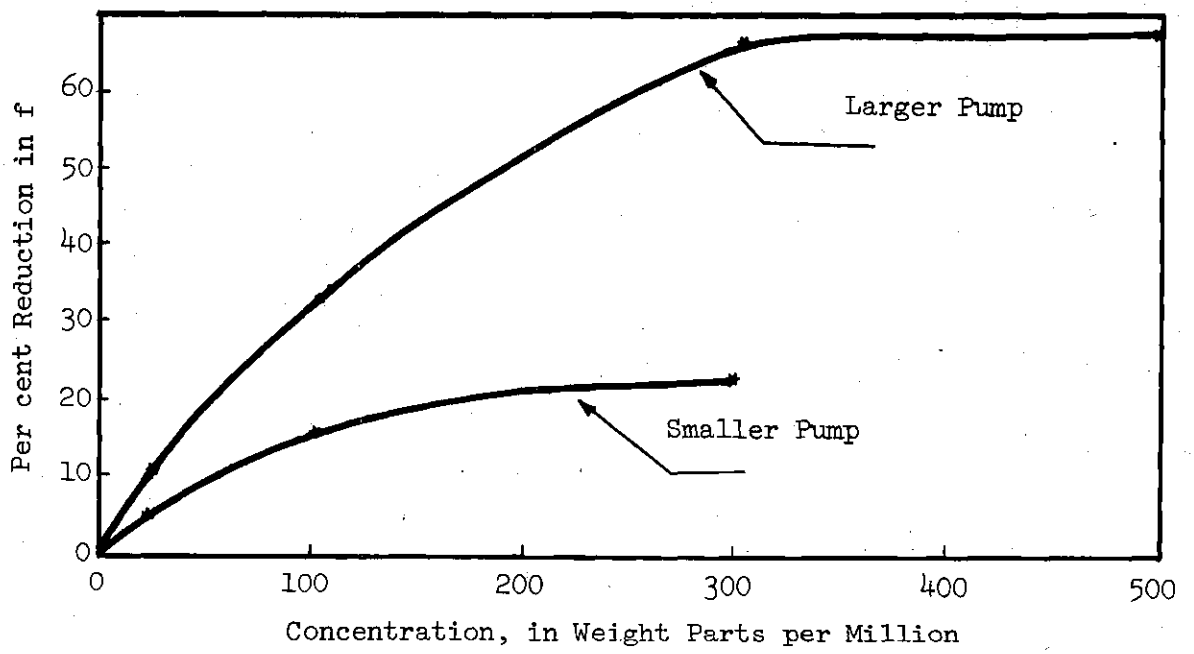


Figure 2. Drag Reduction for Polyhall 295 Flowing in Pipes - Jackson (1)

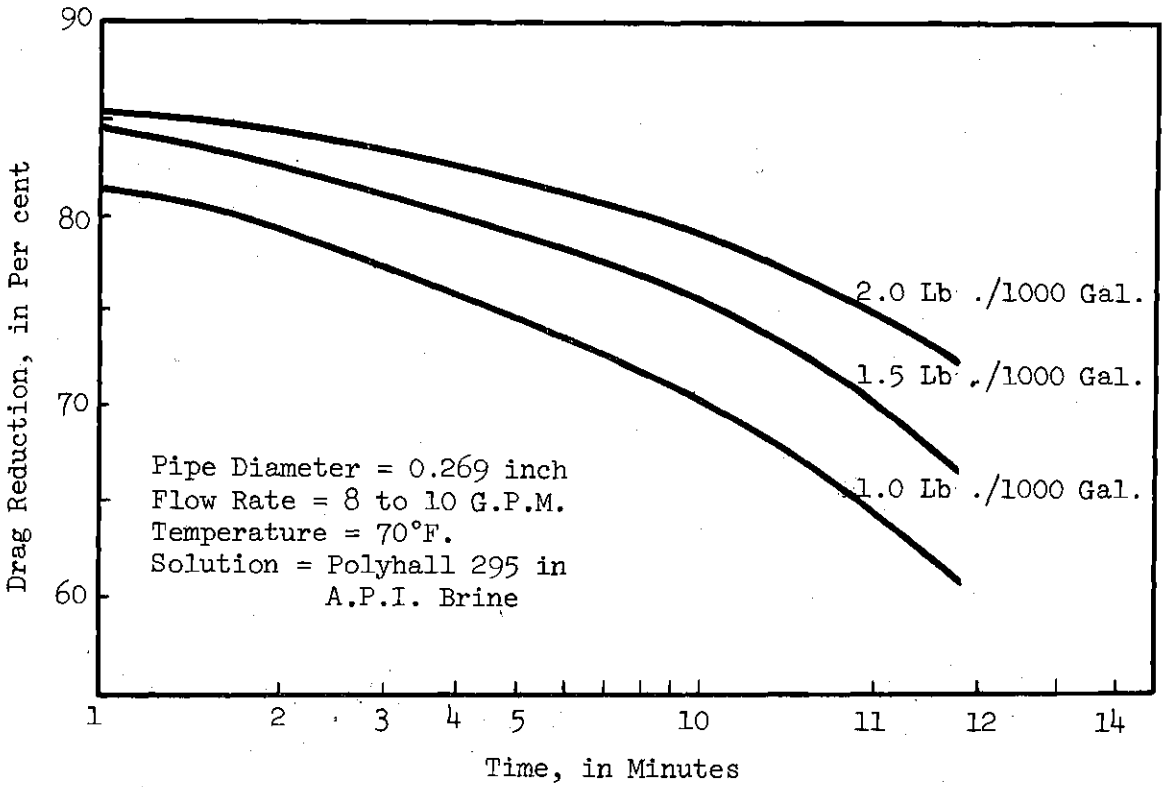


Figure 3. Drag Reduction as a function of Time for Polyhall 295 Recirculating in a Pipe System (18)

flowing at a rate of 8 to 10 gallons per minute.

Flow About Blunt Objects

The reduction of drag on blunt bodies was first reported by Crawford and Pruitt (6) who reported that spheres fell faster in a guar gum solution than in pure water.

To investigate Crawford's and Pruitt's observations further, Ruszczycky (7) in 1965, performed a series of experiments in which he dropped steel spheres, ranging in diameter from 3/8 inch to 1.0 inch, into solutions of guar gum and polyox with very high concentrations ranging from 2,500 wppm to 15,000 wppm. Ruszczycky found that the maximum reductions in the coefficient of drag obtained were 28 per cent for a 1 inch diameter sphere in a 5,000 wppm guar gum solution, and 28 per cent for a 1 inch diameter sphere in a 7,500 wppm polyox solution. Ruszczycky also pointed out that, if this drag reduction were actually due to the apparent viscosity of the polymer solutions, the lowest C_D attainable would be 0.40 at $Re = 5 \times 10^3$. This shift would only account for an 18 per cent reduction from the Newtonian fluid drag coefficient of 0.49 at $Re = 5.19 \times 10^4$ for the 1 inch diameter sphere.

Lang (8), in 1966, measured the coefficients of drag of spheres and other blunt objects falling in 200 and 1,000 wppm polyox solutions. He found that: (a) the maximum drag reduction for spheres was 69 per cent for the case of a 2 inch diameter steel sphere in a 1,000 wppm solution at $Re = 1.4 \times 10^5$, (b) the 200 wppm solution was more effective than the 1,000 wppm solution for reducing sphere drag at Reynolds numbers up to 1.2×10^6, after which the 1,000 wppm solution was more

effective, (c) the concentrated polymer solutions produced a rearward movement of the point of boundary layer separation on the spheres, (d) the additives produced an apparent decrease in turbulent mixing in the near wake, and (e) the addition of polymer produced little or no drag reduction on bodies where the point of boundary layer separation is fixed, such as on cones and disks. The results of Lang (8) for spheres are shown in Figure 4. It should be noted that Lang based all his Reynolds number values on the viscosity of pure water.

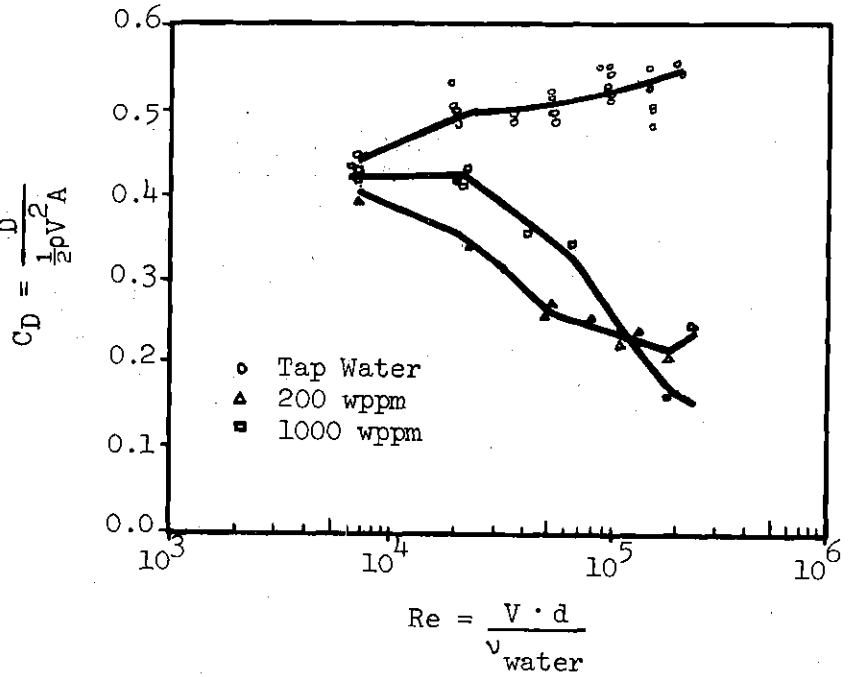


Figure 4. Drag Reduction for Spheres Falling in Polyox Solutions - (After Lang (8))

D. A. White (9) experimented with spheres falling in solutions of Polyox 301 and found that the maximum reduction of the coefficient of drag occurred in a 75 wppm solution, and the per cent reduction in the coefficient of drag increasing values of the Reynolds number, to a

maximum of 45 per cent.

A. White (10) reported on a very interesting phenomenon. He found that for a roughened sphere falling in the turbulent flow regime at a Reynolds number slightly below the critical Reynolds number for a smooth sphere, that a small amount of polymer added to the solution caused the separation point to shift forward. This forward shift caused the wake to increase in diameter and thus increased the form drag. White stated that this forward shift of the separation point was due to a suppression of turbulence in the boundary layer.

Sanders (11), reported in 1967, the results of many experiments which had been performed in the Naval Postgraduate School, Monterey, California, for spheres falling in aqueous poly(ethylene oxide) solutions. These results, combined with those of other experimenters who also studied the fall of spheres in poly(ethylene oxide) solutions (8 and 9), were compared to the results found for poly(ethylene oxide) solutions subjected to turbulent skin drag (5). From these comparisons Sanders drew the following conclusion; as the reduction in drag for flow about a sphere in wake dominated flow is achieved with the same polymers as drag reduction in the turbulent skin friction situation, and as in both cases only a minute amount of polymer was needed to effect the maximum reduction, the mechanism responsible may be the same for both types of flow.

In addition, the following results were obtained; there was no drag reduction with poly(ethylene oxide) for water Reynolds numbers less than 10^4, and for a given concentration, the drag reduction increased with increasing values of Re. Sanders also reported on a very strange phenomenon which occurred in concentrated (> 1000 wppm) polymer solutions. When the spheres were dropped into these solutions, they

accelerated to their terminal velocities, then suddenly changed to a much slower, but once more constant, velocity. This dual terminal velocity phenomenon was found to be a function of a critical shear stress above which the polymer network structure was believed to change.

Gadd (12) performed a series of experiments to learn what effects the polymer additives would have on vortex streets. He found that small concentrations of various polymers made the eddy-shedding more regular than for water, that large reductions in frequency were accompanied by proportionate increases in amplitude, and that there was a tendency for the lower frequency eddies to be favoured at the expense of those of higher frequency.

Vogel and Patterson (14) found by injecting polymer solutions into the boundary layer of a three-dimensional streamlined body that: (a) the drag of the body decreased with increasing molecular weight of the polymer, (b) the drag decreased as the polymer concentration increased, (c) the drag reduction increased for increased flow rates of polymer solution, (d) there was a change in the mean square turbulent velocities in the wake, and (e) the wake exhibited changes in its velocity profile. These experiments were performed in the turbulent flow regime on a torpedo shaped body which was held stationery as the flow moved past it.

CHAPTER III

THEORETICAL CONSIDERATIONS

In order that the results obtained with polymer solutions may be better understood, the fundamental aspects of the flow of an incompressible Newtonian fluid about a sphere will first be reviewed. The flow characteristics for the polymer solutions, as reported in the literature review, will then be compared to the flow characteristics of a Newtonian fluid. These results will then be used to predict the characteristics to be expected with dilute aqueous Polyhall solutions.

Newtonian Flow About a Sphere

The forces acting on a sphere falling in an incompressible Newtonian fluid have been expressed in terms of a curve of the dimensionless drag coefficient versus the Reynolds number (See Figure 5). The drag coefficient for spheres is defined as the drag force, F_D, divided by the dynamic pressure, $\frac{1}{2}\rho V^2$, multiplied by the projected frontal area of the sphere, A. The Reynolds number is a dimensionless quantity relating the inertial forces to the viscous forces acting on the sphere. The drag coefficient, C_D, and the Reynolds number, Re, are expressed as:

$$C_D = \frac{F_D}{\frac{1}{2}\rho A V^2} \tag{4}$$

and

$$Re = \frac{\rho V d}{\mu} = \frac{V d}{\nu} \tag{5}$$

in which F_D = Drag force

A = Projected area perpendicular to flow

V = Velocity of the sphere

d = Diameter of the sphere

ρ = Fluid mass density

μ = Shear viscosity

ν = Kinematic viscosity

For non-Newtonian fluids, the Reynolds number must be carefully interpreted, as the concept of viscosity does not appear to be as meaningful for these fluids.

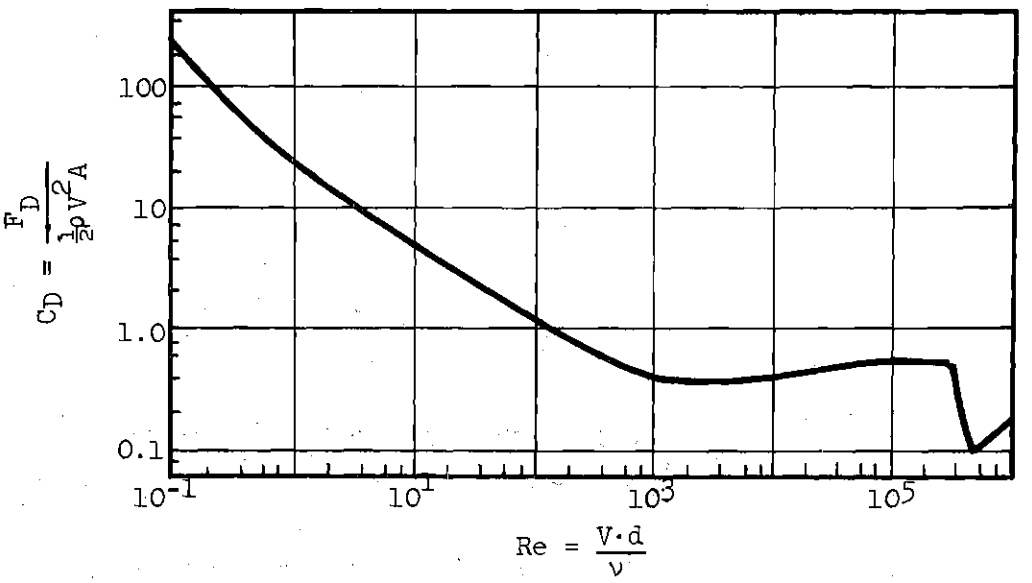


Figure 5. Drag Coefficient vs. Reynolds number for Spheres in Free-fall in a Newtonian Fluid. (After Schlichting (17)).

Flow Configuration as a Function of the Reynolds Number

For Re < 1, the relation between the drag coefficient and the Reynolds number is available in the form of Stokes' law, $C_D = 24/Re$, for Newtonian fluids. This law is based on an approximate solution to the Navier-Stokes equations in which the convective acceleration terms are considered negligible in comparison with those of viscous shear, and are thus ignored completely. As the Reynolds number increases past one, there is a steady trend away from the theorized flow of Stokes' law towards a boundary layer flow, with separation beginning to take place at the rear stagnation point of the sphere at about Re = 20. As the Reynolds number continues to increase, boundary layer flow continued to develop until Re is about 10^3. For $10^3 < Re < 10^5$ a laminar boundary layer exists with separation occurring near the equator. In this region, the wake is nearly independent of Re, resulting in a constant value of C_D, and the drag is proportional to the velocity squared.

At a Reynolds number of approximately 3×10^5, the "critical Reynolds number," the diameter of the wake decreases, resulting in a sudden decrease in C_D. This sharp change in the flow pattern is brought about by the transition from laminar boundary layer flow to turbulent boundary layer flow, and is discussed below.

Separation and Drag

In frictionless flow, a fluid particle at D in Figure 6, will accelerate to E and then decelerate to F as it moves downstream along any streamline. Due to the conservation of momentum, this acceleration and deceleration will have as its counter-part a corresponding negative pressure gradient from D to E, and a positive pressure gradient from E

to F. Since the positive pressure gradient from E to F has the effect of opposing the motion of fluid particles moving downstream along the sphere, it is often called the "adverse" pressure gradient. As long as there are no dissipative viscous forces in the system, a fluid particle traveling from D to F will reach F with the same momentum it had at D.

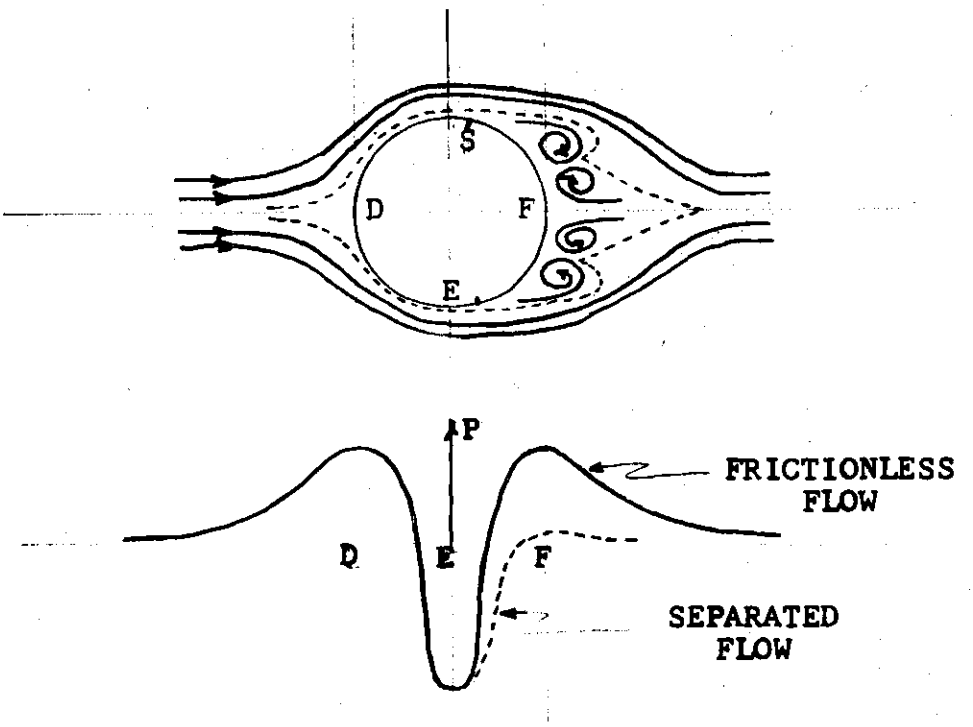


Figure 6. Flow and Pressure Distribution about a Sphere Moving in a Newtonian Fluid.

In a real fluid however, much of the kinetic energy which a fluid particle would gain in going from D to E in frictionless flow, is spent in opposing the large friction forces in the thin boundary layer. Such a particle therefore does not have enough momentum to oppose the positive pressure gradient from E to F, and comes to a stop somewhere

between these two points, a condition known as "stall." This stalled particle, owing to the "adverse" pressure gradient, will then start to move in a direction opposite to that of the main flow (i.e. from F to E). The interaction of the two flows, backflow along the body and forward flow outside the layer of backflow, results in a vortex eddy. As this eddy is between the sphere and the boundary layer, the boundary layer is forced to separate from the sphere. When the boundary layer separates, the pressure at F is greatly reduced causing a net pressure force which opposes the motion of the sphere. This net force is referred to as the pressure drag.

In the laminar boundary layer flow discussed above, there is no mixing perpendicular to the streamlines. Thus, the only energy which a fluid particle in the boundary layer has is that which is impressed upon it by the free stream pressure distribution. In turbulent boundary layer flow, however, there is momentum transfer across streamlines. This additional momentum, transported into the region close to the sphere, helps a fluid particle in the boundary layer to surmount the pressure hill between E and F thereby delaying separation and reducing the wake diameter. As the pressure recovery at F is a function of the wake diameter, being greatest when the wake diameter is very small, the pressure recovery will be greater in turbulent boundary layer flow than in the separated laminar boundary flow. The result is a reduction of the pressure drag on the sphere, and thus a reduction of the total drag.

Classes of Polymers

The additive polymers which have been investigated (5) for their drag reducing effects fall into three main classes:

a. random-coiling polymers

b. linear polymers

c. rigid or compact polymers

Of these three types, those of group (a) and group (b) have exhibited the most impressive drag reducing effects.

Random-Coiling Polymers

A random-coiling polymer is one in which the macromolecules assume an expanded ball like shape containing mostly solvent in dilute solutions. These balls range in diameter from 300 Å to 6000 Å depending on molecular weight, polymer configuration, and effectiveness of the solvent. Their exceptional drag reducing effects appear to be due to their ability to expend energy, thus dampening out some turbulence, when elongated by a high shear stress. This dampening effect increases the boundary layer stability which causes a decrease in drag, for example, the dampening out of turbulent spots would greatly increase boundary layer stability. This energy is expended in keeping the molecule away from its coiled equilibrium configuration. One polymer of this type of poly-(ethylene oxide).

Linear Polymers

A linear polymer, as its name implies, is one which assumes an elongated configuration in dilute solutions. One class of these polymers is the polyelectrolytes. These molecules assume their elongated configuration as a result of repulsive forces between adjacent polyions. These polymers are excellent for reducing turbulent friction losses as they act to dampen turbulence throughout the flowing solution, even in regions of minimal shearing stress.

Rigid or Compact Polymers

Polymers such as globular proteins and tobacco mosaic viruses are of the rigid or compact type. These polymers have been found to be relatively ineffective for reducing drag.

Summary of Polymer Solution Behavior

The reduction of turbulent skin friction drag for liquid flow in pipes, with the addition of long-chain polymer to the solution, seemed to be caused by two effects.

The first was a stabilization of the flow in the boundary layer, as depicted by an increase in the value of the boundary layer Reynolds number, Ro. The second was a bulk effect of the polymers to change the velocity profile in the pipe core flow. This second effect occurred only when the concentration of polymer in the solution was so large that the polymer macromolecules formed a continuum in the core region.

Some characteristics of the long-chain polymers used were:

(a) Maximum drag reductions were obtained with very low polymer concentrations (below 100 wppm).

(b) As the concentration was increased past an optimum point, the drag reducing effect of the polymers decreased with increasing concentration.

(c) Solutions of random-coiling polymers exhibited no drag reductions below a threshold Reynolds number.

(d) Linear molecules did not exhibit a dependence upon a threshold Reynolds number for drag reduction.

Theorized Results For Flow About Spheres

Based on the results obtained in the experiments reviewed above, the following results would be expected for flow around spheres:

1. A stabilization of the laminar boundary layer for separated flow about a sphere. This would be characterized by a gradual decrease in the wake diameter as the Reynolds number increases past Re = 10^3. This is in deference to the very sharp decrease in the wake diameter which occurs at the transition from laminar boundary layer to turbulent boundary layer flow for Newtonian fluids.

If the above hypothesis is correct, the decrease in drag should only be discernible in the range of $10^3 < Re < 10^5$.

2. The maximum drag reductions should occur with very low polymer concentrations, and should increase as the concentration after reaching the optimum point.
3. Linear polymers should be capable of reducing the drag at Reynolds numbers below those required for random-coiling polymers. This result, however, would only be apparent if the threshold Reynolds number for the random-coiling polymers is greater than 10^3.

CHAPTER IV

INSTRUMENTATION AND EQUIPMENT

The equipment used in these experiments was designed to enable terminal velocity measurements to be taken for steel and glass spheres falling in polymer solutions, within the limits of the following criteria:

(a) The spheres should be large enough to permit easy visual and photographic sightings.

(b) The terminal velocity of the spheres should be slow enough to permit visual measurement.

(c) Spheres of different materials, and thus different densities, should all fall with the same terminal velocity.

(d) The apparatus should make it possible for the spheres to be withdrawn from the solution after descent with as little disturbance to the solution as possible.

(e) The ratio of sphere diameter to tube diameter should be small to minimize wall effects.

To accommodate the first criterion, it was decided that the spheres should be about 1/2 inch in diameter. Glass toy marbles were found to meet this requirement. The marble chosen for use in these experiments had a diameter of 0.640 inch and a specific gravity of 2.588 at 72°F.

To accommodate criteria (b), (c), and (d), a counterweight and pulley system was employed. This system was to satisfy criteria (b) and (c) as it was now possible to adjust the effective weight of the spheres in solution. Criterion (d) was also satisfied as the string enabled the sphere to be removed from the solution without the insertion of other objects which might disturb the structure of the solution.

Based on the reports of J. S. McNown (15 and 16), for the wall effects to be less than 5 per cent, the ratio of sphere diameter to tube diameter should be less than 0.2. For this criteria to be satisfied, for a 0.640 inch sphere, the tube diameter should be at least 3.2 inches in diameter. Based on the above discussion, the following experimental apparatus was constructed.

The apparatus consisted of three 4 inch OD plexiglass tubes with 1/8-inch thick walls 4.4 ft. in length, attached side by side to a 0.5 inch thick plexiglass base. (See Figures 7 and 8).

Each tube was provided with two airplane pulleys to accommodate the counterweight system.. The pulleys were made of nylon with 1/8-inch deep V grooves, and 3/8-inch ball bearings at their centers. The pulleys were positioned so that the spheres descended along the center line of the tube, and the counterweights plumbed on the release mechanism. This latter arrangement was very important as otherwise the counterweights would swing causing an unevenness in the motion of the spheres.

The release mechanism was designed to allow the spheres at each tube to be released simultaneously. This mechanism was very simple and consisted of three small solenoids attached in series to a 9-volt AC

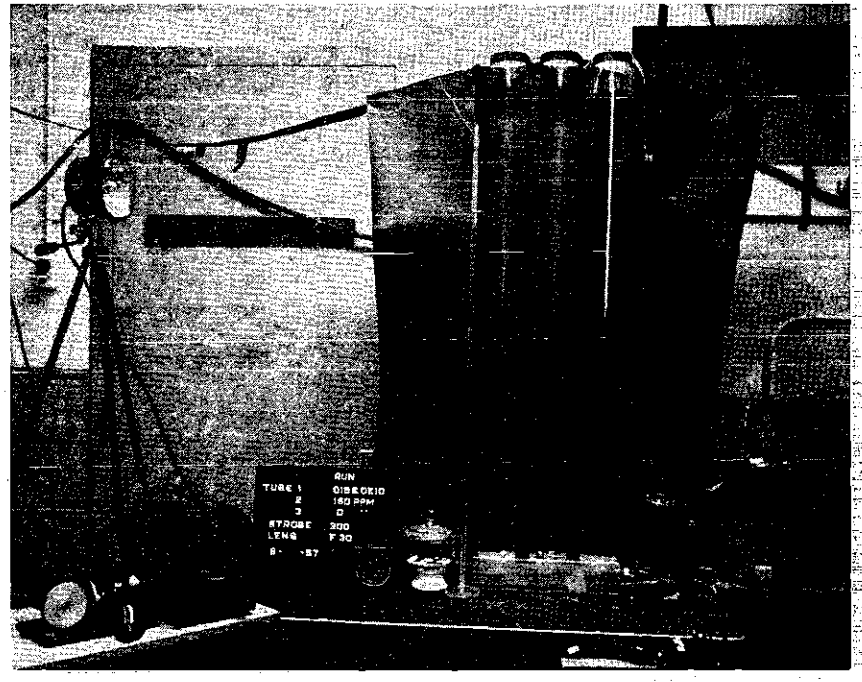


Figure 7. Photograph of Experimental Apparatus

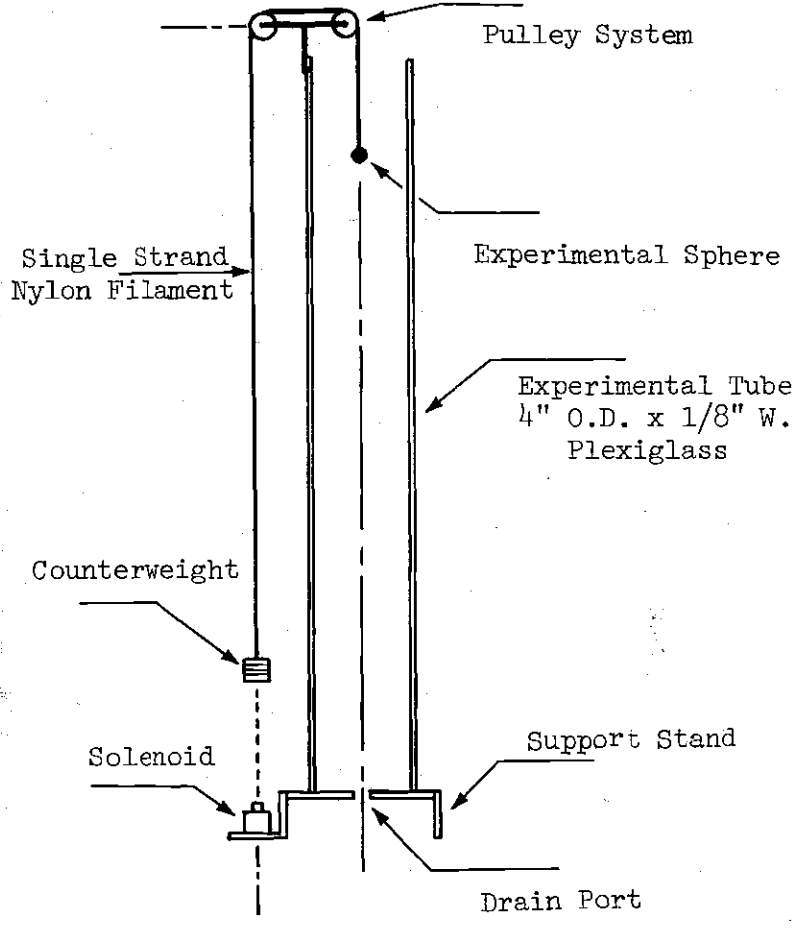


Figure 8. Detail of Experimental Apparatus

transformer. The release time was found to be almost instantaneous with current cut-off for the voltage used.

The spheres and the counterweights were joined by single strand nylon thread approximately 0.006 inches in diameter. A single strand thread was chosen as it would not unravel, imparting spin to the sphere, and nylon was chosen for its strength and durability.

Measurements of terminal velocity were made by determining the time necessary for the sphere to pass two markers one foot apart. For velocities below 0.5 feet per second, the time was determined using a stopwatch. For larger velocities a photographic technique incorporating the use of a strobe light was used to accommodate measurements of displacements and time.

An electric clock with a 4 inch dial permitting readings to within 2/100 of a second was used. The strobe equipment consisted of a high intensity light source capable of flashing from 50 to 1,000 times per minute.

The photographs were taken with a press-view camera using "Polaroid Type #57" black and white film. This film has an ASA rating of 3,000.

Viscosity measurements were made with the "Hoeppler Precision Viscosimeter." The viscosity determined with this apparatus is directly proportional to the time of fall of a ball in a tube. For this experimentation the viscosimeter used had a tube diameter of 15.937mm, and the ball used had a diameter of 15.6295mm and a specific gravity of 2.2267. The above statistics are for 20°C.

CHAPTER V

EXPERIMENTAL PROCEDURE

The Polymer Solution

Properties of Polyhall 295

Polyhall 295 is an anionic copolymer of polyacrylamide, and polyacrylic acid and polysodium styrene sulphonate. The molecular weight of this polymer is approximately 3.8×10^6. Solutions of Polyhall, when subjected to low shear rates, have been reported by Gadd (13) to be markedly shear thinning, that is, the apparent viscosity of the solution tends to decrease with increasing shear rate. This polymer appeared to degrade rapidly when subjected to the high mechanical shear encountered in a centrifugal pump (1).

Addition of Polymer to Solution

When the polymer was sprinkled over the water while the solution was being mixed, the polymer appeared to go into solution in less than five minutes.

When a large number of particles were added at one time, on the other hand, a tough skinned aggregate was formed. This mass dissolved and dispersed very slowly, often taking several hours to disperse completely. Thus, the particles of polymer were added to the water in the experimental tubes very slowly with constant mixing.

Air Entrainment

An additional problem encountered in the polymer solution was

air entrainment. Air bubbles were always trapped in the solution when polymer was added or when the solution was mixed. Air bubbles in solution created two undesirable effects: first, the bubbles tended to stick to the spheres causing different flow patterns, and second, air bubbles in the path of a falling sphere reduced the fluid bulk density, thereby increasing the settling velocity. This reduction of density would also cause the experimentally determined coefficient of drag to be lower.

The entrained air took the form of small air bubbles approximately 0.05 in. in diameter. For example, these bubbles took two hours to rise out of solution for polymer concentrations of 50 wppm and six hours for concentrations of 350 wppm.

One observation worth noting occurred shortly after the polymer was added to the solution. The non-diffused polymer molecules formed longitudinally aligned wiry masses, sometimes as long as 3 cm. These agglomerations were made visible by air bubbles which attached themselves to their extremities, and by rising air bubbles which had their paths obstructed by them. After a few hours (after the molecules had diffused) this phenomenon was no longer evident.

To avoid the problems of clumped masses and of air entrainment, and in order to assure that the polymer had diffused completely, the polymer was added to the solution some 16 hours before experimentation was to take place.

Experimental Procedure

The experiments performed in this study were designed to determine three characteristics of aqueous Polyhall 295 solutions:

First, to what extent, if any, the addition of Polyhall 295 to water altered the flow characteristics for flow around blunt objects from those usually found with Newtonian fluids. This was studied by measuring the terminal velocities of counterweighted spheres falling in aqueous Polyhall 295 solutions of 25, 50, 100, 150, 200, 250, and 350 wppm concentration.

Second, to determine to what extent aqueous Polyhall 295 solutions were degraded by time and usage. This was done by taking measurements of the apparent viscosity of the polymer solutions at various stages throughout the experiments.

Third, to determine if some polymer is adsorbed on the surface of a sphere, and what effect this residue might have on subsequent experiments with the same sphere. This particular study was performed by taking spheres out of polymer solutions, and dropping them in experimental tubes filled with distilled and deionized water. This was done for two situations; for one test, a sphere was dropped into distilled water while still wet with polymer solution, and in the other test, a sphere was dropped into distilled water after being placed overnight in a desiccator. A second part of this experiment was performed to determine if polymer encrusted on the sphere had any effect on drag. In all cases, the effect of the polymer was determined by measuring the terminal velocity of a test sphere and comparing it with the terminal velocity found for the same sphere when thoroughly cleaned and falling in

distilled water.

The terminal velocities were determined by measuring the time it took for a sphere to pass two markers on the experimental tubes, one foot apart, with a stop watch. For cases where reflexes were judged to be not sharp enough to accurately determine the time elapsed, falling spheres were photographed using stroboscopic illumination, with the flash rate set at five flashes per second. Stroboscopic photographs were also employed to insure that the terminal velocity had been reached.

During the early part of these experiments it was found that when a sphere was dropped repeatedly into a tube of polymer solution, the terminal velocity of the sphere increased with each successive drop. This effect may have been due to one of two things:

(a) The physical structure of the polymer solution had been changed by the moving sphere, or

(b) Polymer had been adsorbed on the sphere, and was causing the sphere to behave differently.

A possibility exists, of course, of both phenomena taking place simultaneously.

To determine which of these phenomena had in fact occurred, two additional experiments were performed. The first consisted of taking the sphere out of the tube after six drops, washing it thoroughly, placing it back in the same tube, and dropping it in the same flow path, determining the terminal velocity for each additional drop. The second experiment was the same as the first, except that the polymer solution was re-mixed while the sphere was being dropped.

CHAPTER VI

DISCUSSION OF RESULTS

As described in the previous chapter, the experiments performed in this study consisted, for the most part, of measuring the terminal velocities of steel and glass spheres falling in tubes of either distilled and deionized water or aqueous "Polyhall 295" solutions, and of measuring the apparent viscosities of these solutions.

Calibration of Apparatus

Experiments were performed to determine the relationship between the drag coefficient and the Reynolds number for Newtonian fluids in the apparatus of this study. The Newtonian fluids used were various concentrations of ethylene-glycol in water. The results of these experiments have been plotted in Figure 9. Also shown in this figure are the values of C_D vs. Re determined by the experimenters of References (15 and 16) for the free fall of spheres in tubes whose diameters were six times that of the spheres, the same diameter ratio used in this study. The differences in the measured coefficients of drag for the various Reynolds numbers of these two curves were probably due to differences in the physical makeup of the different systems. Some of the factors which might possibly have affected the calculated drag on the spheres in this system were: the inability of the sphere to rotate freely as it was attached to a taut thread, and the possibility that a systematic error was introduced in the calculation of the drag force and the pulley

friction force. As the rotation of the pulleys was as slow as 1/2 revolution per second for the smaller terminal velocities, the increased resistance to motion, experienced at the slower sphere velocities was due to increased forces in the pulley system. In order that the results of these experiments might be compared to flow about spheres for Newtonian fluids and to the results found with other polymers in similar experiments, the values of the coefficient of drag were adjusted to take into account the varying drag force. This adjustment was in the form of a drag force which varied with velocity, and was determined by adjusting the value of the pulley friction force to match the net weight force so that the plot of drag coefficient vs. Reynolds number for a Newtonian fluid in the apparatus of this experiment would coincide with the curve plotted by McNown (15 and 16) for free falling spheres in Newtonian fluids. The correction factor curves are shown in Figure 10. The fact that the drag vs. velocity curves of the glass and steel spheres appear to coincide is not surprising, as the counterweights were adjusted so as to make the two spheres fall with almost the same velocity in distilled water. However, the result that the friction factor is greater for the steel sphere than for the glass sphere is significant as it adds support to the deduction that the additional drag was due to pulley friction. That is, the steel sphere system weighed more than three times that of the glass sphere system, and thus, the total force acting on the pulleys was greater.

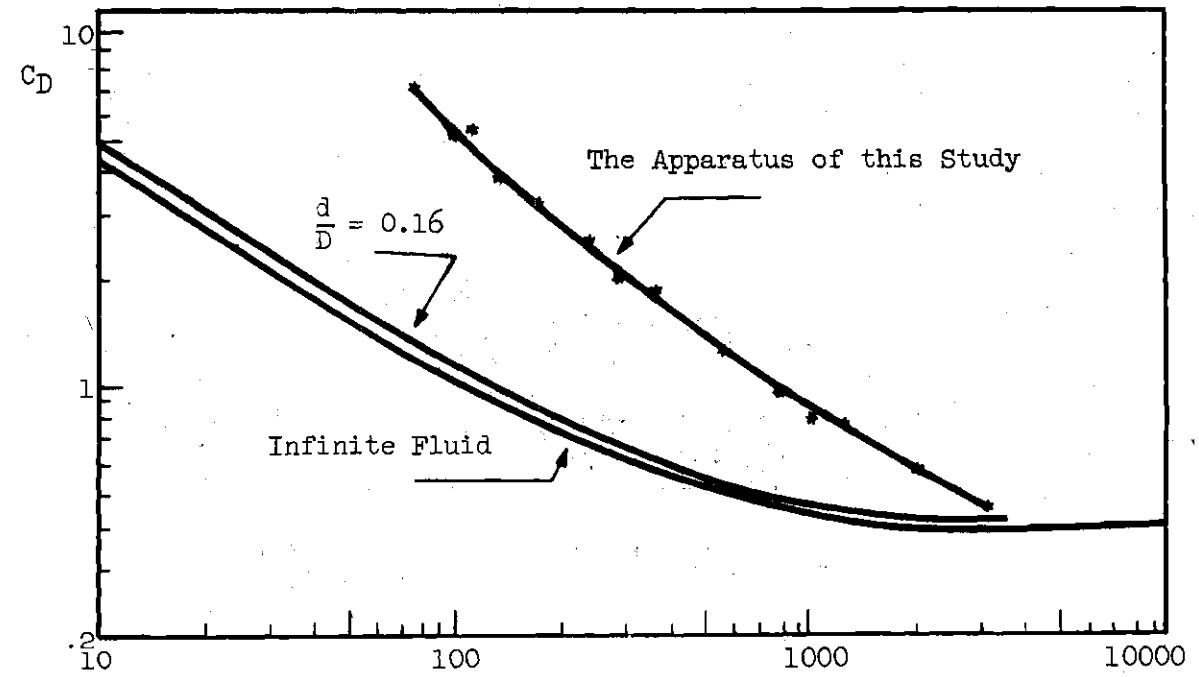


Figure 9. Drag Coefficient vs. Reynolds number for a Newtonian Fluid in the Apparatus of This Study

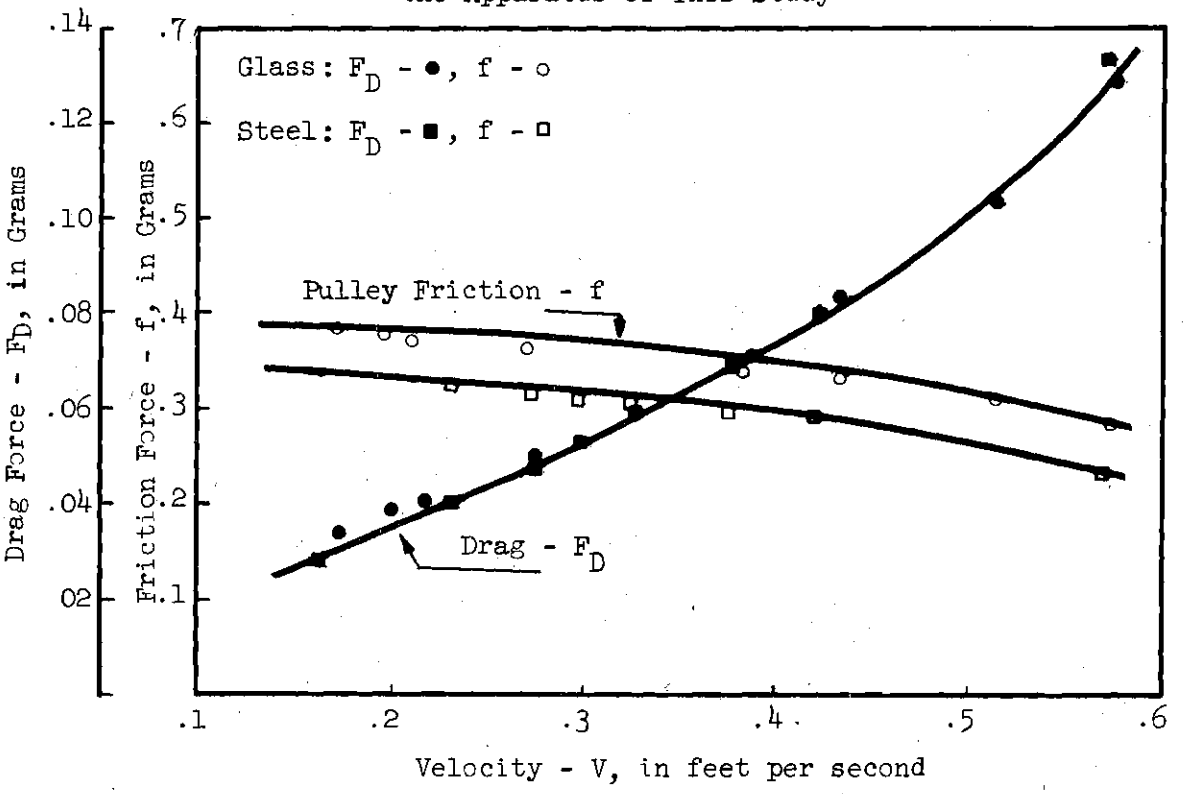


Figure 10. Pulley Force Correction Curve for the Apparatus of This Study

Reynolds Numbers in Polyhall* Solutions

Aqueous Polyhall solutions do not have a Newtonian viscosity, but rather, they exhibit an apparent viscosity which is a function of the shear being applied to the solution. The use of a measured value of the apparent viscosity as obtained by means of a viscosimeter, in determining the Reynolds number for flow in polymer solutions, would yield erroneous results unless the viscosity measurements could be taken at the same shear rate as that encountered in the particular experiment.

Other experimenters (5, 8, and 11) have computed the Reynolds number on the basis of a pure water viscosity. They argued that, for the polymer they were using, the apparent viscosity of their polymer solutions was found to be very close to that of pure water. From the plots of Polyhall 295 concentration in water vs. the apparent viscosity (See Figure 13) it was obvious that this same assumption could not be made for Polyhall for concentrations greater than 25 wppm.

The Reynolds numbers used in this experiment were calculated using the viscosity of pure water, but not for the reason discussed above.

An engineering application in which the results of these experiments might be used is the reduction of drag on naval vessels. In this type of application the engineer would be interested in learning how the addition of polymer to the water affected the ships velocity with the ships engines supplying a constant drive. Similarly for falling sphere

---

*For the remainder of this discussion, the term Polyhall should be taken as meaning Polyhall 295 unless otherwise noted. Other grades of Polyhall are commercially available.

experiments, the force causing the sphere to fall is a constant equal to the weight of the sphere minus the buoyant force. The results the engineer is looking for is how the terminal velocity of the sphere varies with polymer concentration (19).

Experimental Results

As explained in the chapter on experimental procedure, three distinct experiments were performed with aqueous Polyhall 295 solutions in this study. The first experiment was designed to see how the terminal velocity of counterweighted spheres varied with concentration of polymer. The second studied the extent to which Polyhall 295 was subject to degradation. The third was devised to see if the polymer would be adsorbed on the surface of the spheres, and what effect this would have on subsequent experimentation with the same spheres.

Terminal Velocity and Polymer Concentration

To determine how the addition of Polyhall 295 to water affected the drag on spheres, 5/8 inch steel and glass spheres were dropped into aqueous Polyhall solutions of various concentrations. The results of these experiments are shown in Figures 11 and 12.

The two velocities shown for each concentration are a function of the polymer being aligned in solution to the direction of the moving sphere, thus offering less resistance to motion. This effect, which is discussed later in this chapter, should not be confused with the dual terminal velocity phenomenon reported by Sanders (11), or with polymer degradation.

For the points on Figure 11 no distinction has been made between

the steel and the glass spheres, as these spheres were used interchangeably throughout this experiment, and as the velocities of both spheres were the same within 1.1 per cent when falling in pure water, (for example, in one test the time to fall 1 foot in pure water was for these spheres: Steel - 1.74 sec., Glass - 1.76 sec.). The plotted points were chosen to represent the same number of sphere drops to effect the same apparent polymer realignment and degradation if any. Worthy of note is the fact that the more concentrated solutions were subject to greater changes due to this realignment. In the curve of Figure 12 these same points have been plotted using the correction curves of Figure 10 to calculate the drag coefficients. The Reynolds number used is based on the viscosity of water.

Also apparent from Figures 11 and 12 is a decrease in the drag from that with a Newtonian fluid, which is affected for Polymer concentrations below 50 wppm. As the concentration rises above 50 wppm, the drag increases with increasing polymer concentration. This result is perhaps similar to that found for Polyhall solutions flowing in pipes (See Figure 1). The difference was that in pipe flow the drag did not rise above that of a Newtonian fluid above 50 wppm concentration although there was a relative increase in the drag. One possible explanation for this apparent difference in the effect of polymer on the flow is that, for the Reynolds numbers encountered in these experiments, the shear rates might not have been high enough to overcome the effects of the increased viscosities of these solutions. To test this hypothesis, a few points were plotted on the C_D vs. Re curve using the apparent viscosities measured for these solutions in calculating the Reynolds number.

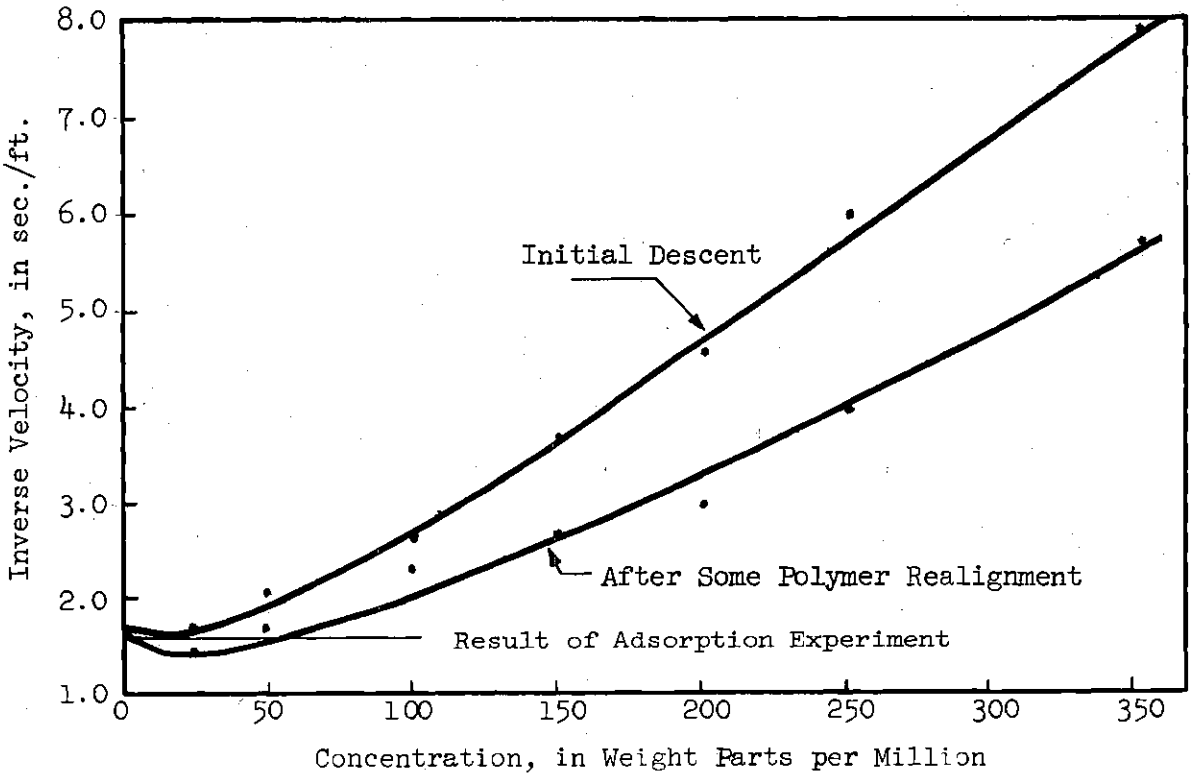


Figure 11. Terminal Velocity vs. Polymer Concentration for Spheres Falling in Aqueous Polyhall 295 Solutions

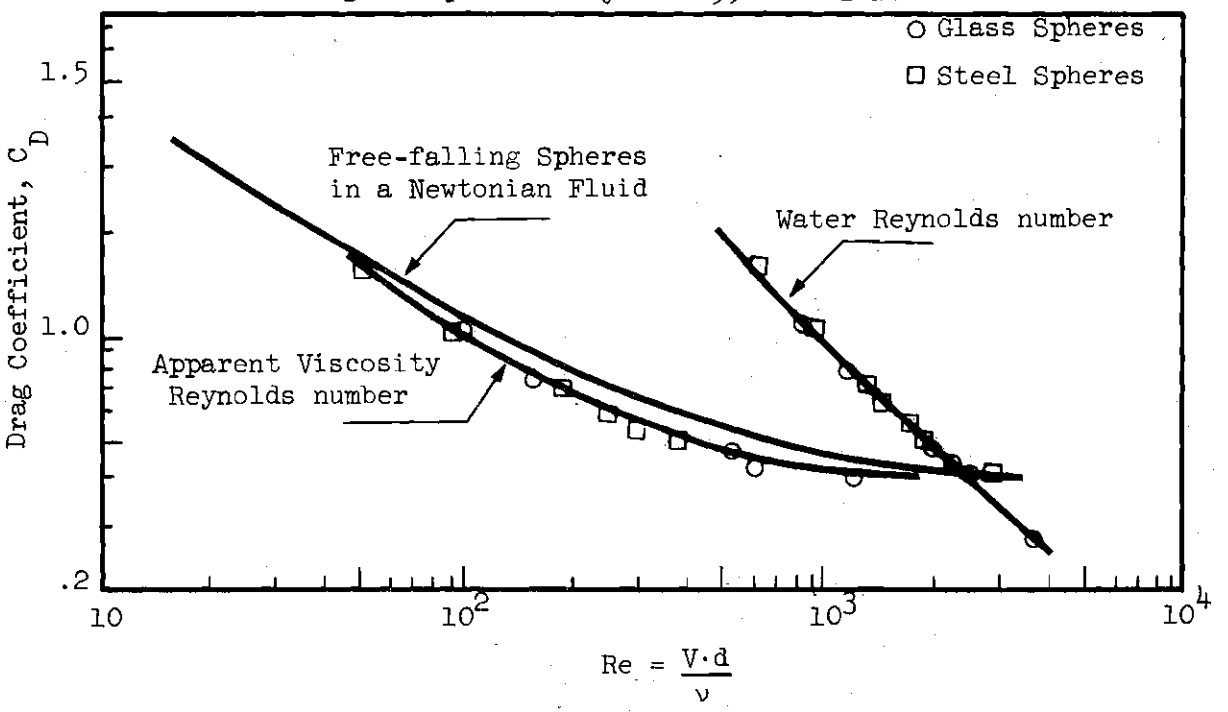


Figure 12. Drag Coefficient vs. Reynolds Number for Spheres Falling in Aqueous Polyhall 295 Solutions

This curve is shown in Figure 12. The fact that this curve happens to fall below that for a Newtonian fluid can not be used to conclude with certainty that there is a reduction of drag, as the Reynolds numbers thus determined, it must be remembered, were based on the apparent viscosities of the solutions.

The greatest decrease in the drag coefficient was found for the 25 wppm solution. This solution produced a drag coefficient of 0.27 at a water Reynolds number of Re = 3760 and represents a drag reduction of 36 per cent over that of a Newtonian fluid. Significant is the fact that this drag reduction was found for a Reynolds number below 10^4, the value of Re below which Sanders (11) and White (9) reported no drag reductions with poly(ethylene oxide). This result seems to indicate that Re = 10^4 may be a threshold value for the random-coiling poly(ethylene oxide) solutions.

Realignment Experiments

When the spheres were repeatedly dropped into a polymer solution, the terminal velocities of the spheres were found to increase with each successive drop. This relative increase was most apparent between the first and second drop and reduced to almost zero by the tenth drop. The percentage increase of the terminal velocity between the first and second drops was also found to increase with polymer concentration, being 13.6 per cent for the 250 wppm solution and only 5.8 per cent for the 150 wppm solution.

An experiment was performed to determine whether this increase in velocity was due to some chemical effect of the polymer being adsorbed on the sphere, or some effect that the falling sphere had on the

structure of the polymer solution. This experiment consisted of (a) dropping the spheres in the polymer solutions six times, (b) removing them from the tubes and washing them thoroughly in pure water, sometimes mixing the solutions in the interim and sometimes not, and (c) replacing the spheres in the tubes and taking further terminal velocity measurements. From this experiment it was determined that just washing the sphere had no effect on the terminal velocities. When the tubes were stirred, however, the terminal velocities of the spheres decreased to a value found initially with about the third successive drop. This result indicated that the increase in terminal velocity measured was definitely caused by some restructuring of the polymer solution. Considering that Polyhall 295 is a linear polymer and is thus normally extended in solution, it was hypothesized from this experimenter that the sphere falling always in the same path (due to the pulley system) caused the polymer molecules to align themselves in the direction of flow. This supposition was substantiated by Alter (18).

Degradation Experiments

A series of experiments were devised to determine how aqueous solutions of Polyhall 295 degraded with both mechanical shear and time, particularly, for the degree of mechanical shear experienced by the solutions in the experiments discussed above. For these experiments, the apparent viscosity was chosen as a good measure of polymer degradation. These apparent viscosities were measured with the Hoeppler viscosimeter described earlier in this paper.

The results of the first part of this experiment are shown in Figure 13 as plots of apparent viscosity vs. polymer concentration for

different degrees of mechanical degradation. The four curves shown are for the conditions described below:

"No Shear" - These solutions had been given 12 hours, after polymer addition, to effect diffusion of the polymer throughout the water. These solutions had not been subjected to any form of mechanical mixing aside for a slight swishing of the beaker in which the polymer was diffusing.

"Very Little Shear" - The samples tested for this curve were taken from the experimental tubes at the beginning of the above described experiments. The polymer in these tubes had been given 16 hours to diffuse during which time the solutions were mixed three times to insure complete diffusion in the large tubes.

"Little Shear" - The samples used for this curve were taken two hours after those of the "Very Little Shear" curve. During this time the solutions had been mixed well three times. This curve represents approximately the maximum amount of degradation experienced by the polymers in the experiments described above.

"Excessive Shear" - These points were taken from a plot of apparent viscosity vs. concentration made by Jackson (1). These solutions had been passed many times through a centrifugal pump before their apparent viscosities were measured.

The curves of Figure 13 show that the degree of degradation of Polyhall 295 in water solutions increases as the concentration for a given amount of shear. It is also evident that the shear rates encountered in this study were much less than those encountered by a polymer solution being passed through a recirculating pipe system and

a pump.

A second experiment was performed to determine how time affected the viscosity of a Polyhall 295 solution. Viscosity measurements were taken of samples from within: 3 inches of the top (T), 2 inches of the middle (M), and 5 inches of the bottom (B) of a tube filled with a 350 wppm concentration solution, at four different times. The results of this experiment are shown in Table 1 and indicate that the polymer solution degraded with time.

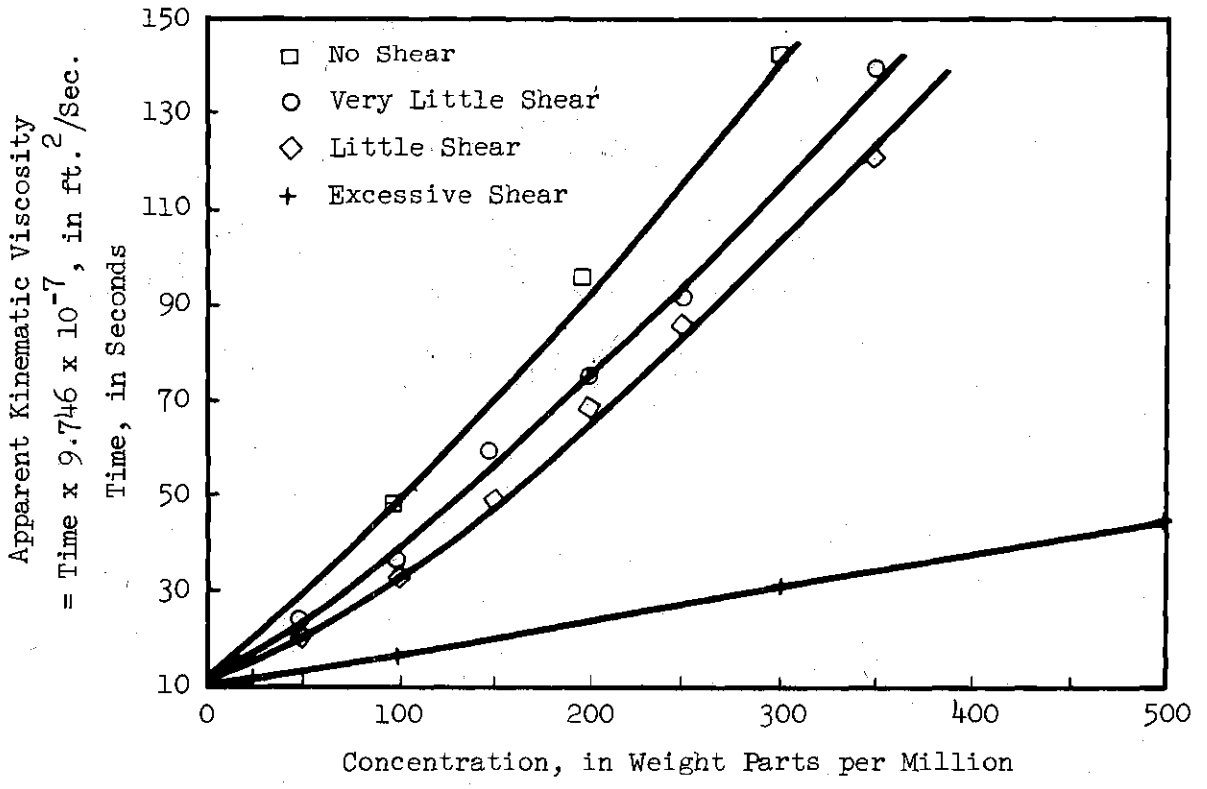


Figure 13. Degradation of Polyhall 295 Solutions as a function of Mechanical Shear and Polymer Concentration

Table 1. Variations in the Apparent Viscosity of a Polyhall 295 Solution

SAMPLING		VISCOSITY MEASUREMENT		
Hour	Sample	Hour	Temp.	Time
00:00*	A - T	02:16	72.5° F	176.92
	A - M	02:33	71.0	138.40
	A - B	02:49	71.7	111.25
02:00	B - T	03:01	71.9	138.31
	B - M	03:17	72.5	131.25
	B - B	03:33	73.5	124.75
73:45	C - T	73:54	73.7	113.70
	C - M	74:06	73.7	114.58
	C - B	74:22	73.7	119.80
73:47	D - T	81:00	73.9	112.70
	D - M	81:17	74.5	107.80
	D - B	81:27	74.9	112.65

Time x 9.746×10^{-7} = Kinematic Viscosity in $Ft^2/Sec.$

*00:00 = Start of Experiment

Sample A - T = Sample taken from the top of the tube for Condition A.

Conditions: A - Polymer added to solution with regular mixing immediately before sample

B - Polymer solution stirred 3 times

C - Three days later, no additional stirring

D - Sample stirred on third day.

Adsorption Experiments

A series of experiments were performed to determine what effect the adsorption of polymer on the spheres might have on the coefficient of drag of the spheres falling in pure water. The first experiment consisted of taking the spheres out of a polymer solution and immediately dropping them several times into a tube filled with distilled and deionized water. The first time the spheres were dropped into the water, they fell with the same velocity as when they were washed and cleaned. With each successive run, however, the terminal velocities increased until the fifth run after which they remained fairly constant. Furthermore, the increase in terminal velocity was independent of the concentration of polymer solution. These data are presented in Figures 14a and 14b where a and b denote steel and glass spheres respectively. The two curves of these figures represent the values of the terminal velocities of the spheres the first time they were dropped into the pure water and the sixth time they were dropped into the tube of water.

The result that the terminal velocity of the initial run was the same as that for a clean ball falling in pure water indicated that the effect of polymer in the boundary layer was negligible. The increase of the terminal velocity with each successive run was thus considered to be due to the polymer being washed off the sphere forming a very dilute solution in the path of sphere. To test this hypothesis the tube of water was stirred - the hypothesis being that the polymer which washed off the sphere remained in a narrow path along the experimental tube. When the solution was mixed, this small amount of polymer was dispersed throughout the entire tube, or into a volume many times that

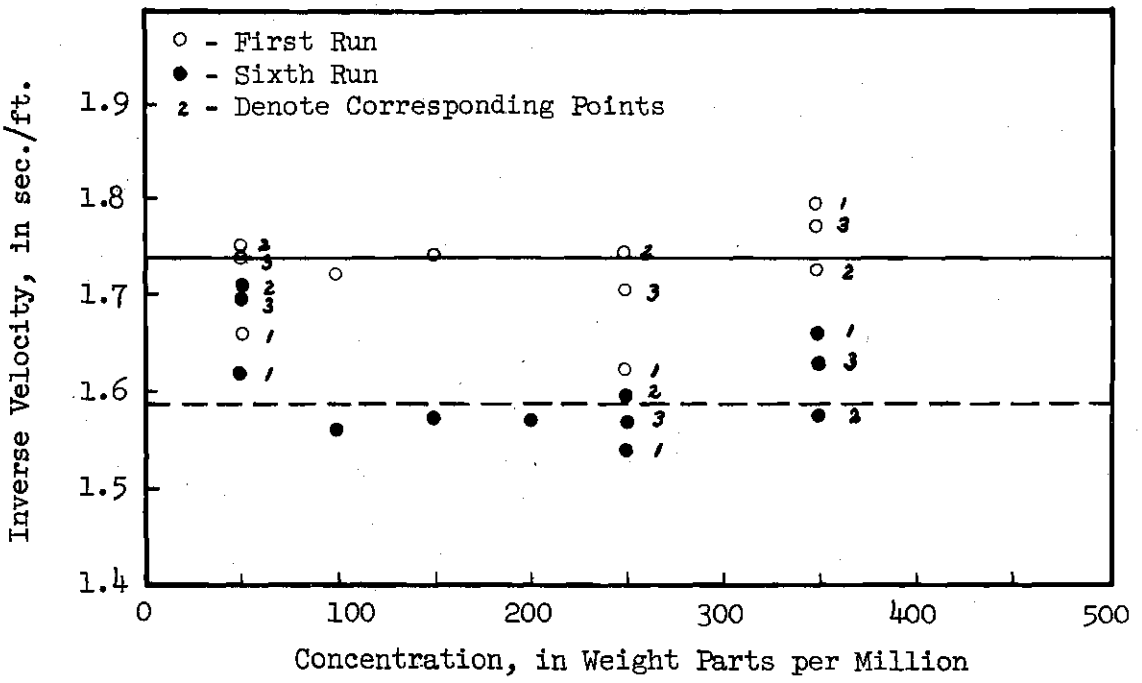


Figure 14a. Steel Spheres

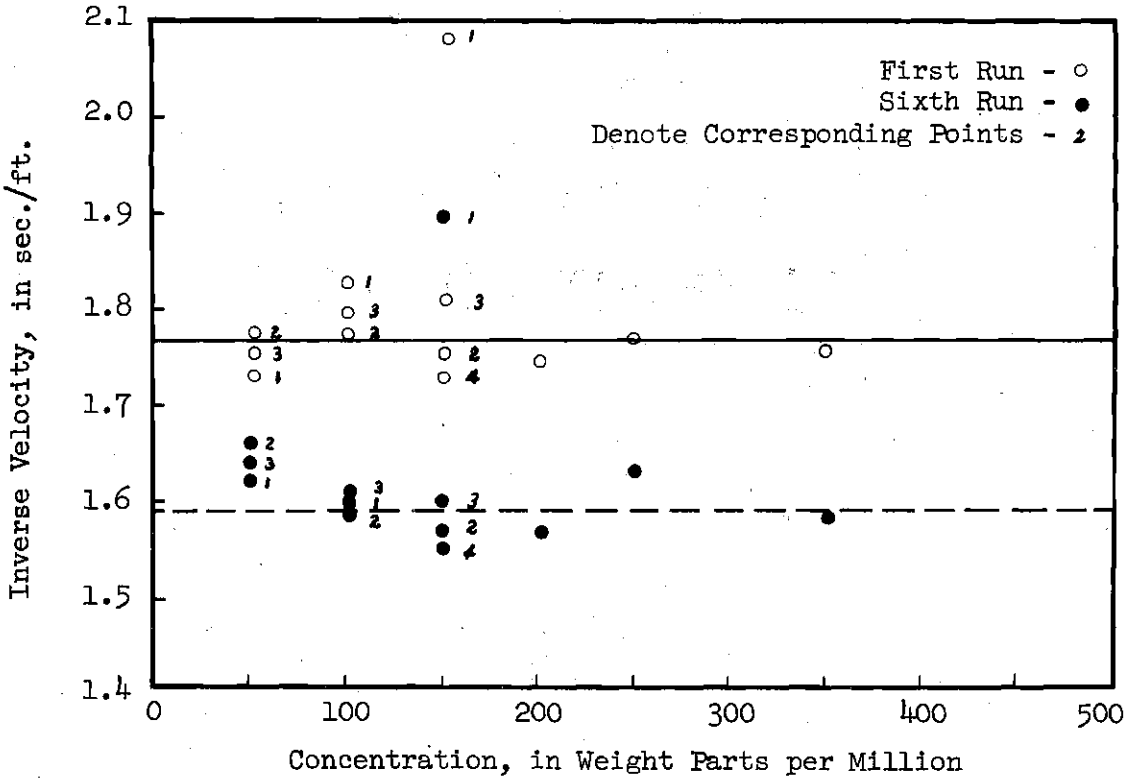


Figure 14b. Glass Spheres

Figure 14. Effect of Polymer Adsorption on the Settling Velocity of Spheres in Pure Water

of the hypothesized path. The result of this study was that after the solution in the tube had been stirred, the terminal velocity decreased to the value found for a clean sphere in distilled water and did not increase with each successive run. This was indicative of the fact that all the polymer had been washed off the sphere in the initial six runs. In addition, the facts that: (a) the terminal velocity increased with each successive run until all the polymer washed off the sphere, and (b) after mixing no effect on the drag was discernible, seem to indicate that for very dilute solutions, the percentage of drag reduction increases with polymer concentration.

A second experiment of this series was performed to determine what the effect would be of allowing adsorbed polymer on a sphere to dry. Spheres which had been in polymer solutions were placed in a desiccator to dry for approximately 17 hours. At the end of this period, the spheres were dropped into a tube filled with distilled and deionized water and their terminal velocities measured. The results of this experiment have been plotted in Figures 15a and 15b and can be summarized as follows: (a) the drag on the spheres showed an increase over the drag in pure water of a clean sphere the first time they were dropped into the water, (b) the drag on the spheres decreased with each consecutive run, approaching the values obtained in the previous experiment by the sixth run, and (c) the drag of the initial run appeared to increase with the drying time. To verify that the drag was proportional to the drying time, a further simple experiment was performed. In this experiment steel and glass spheres were: (a) placed in polymer solutions of various concentrations, (b) placed in a desiccator to dry for 22, 46,

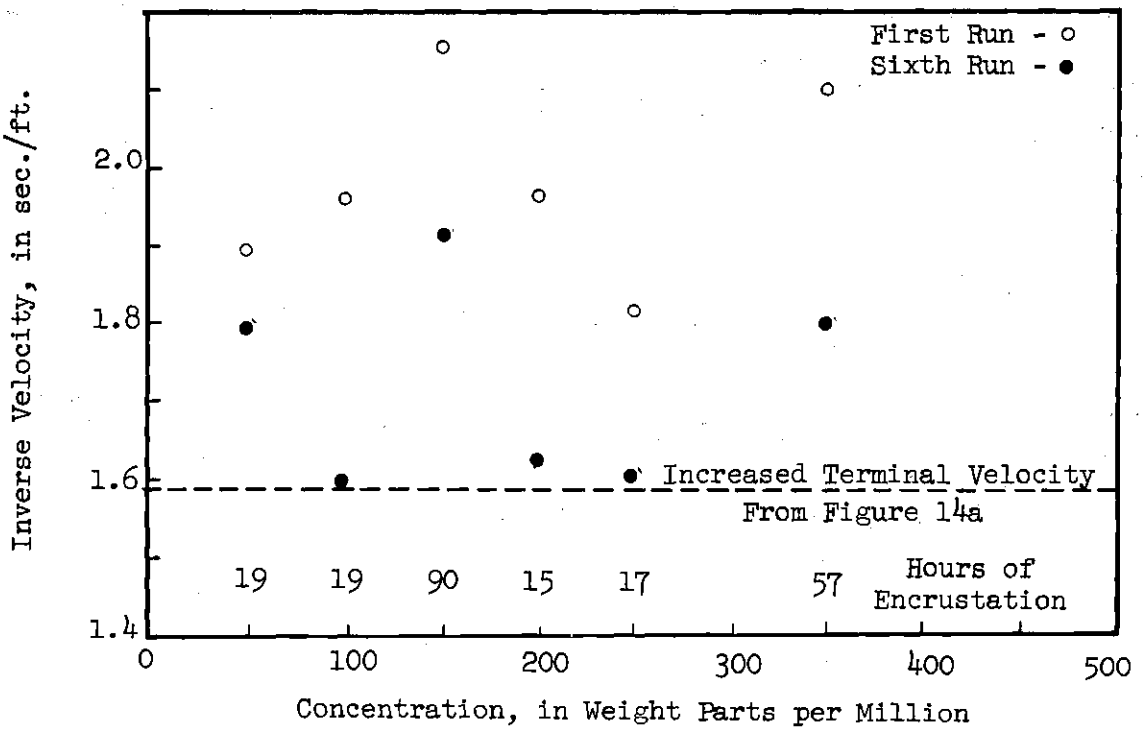


Figure 15a. Steel Spheres

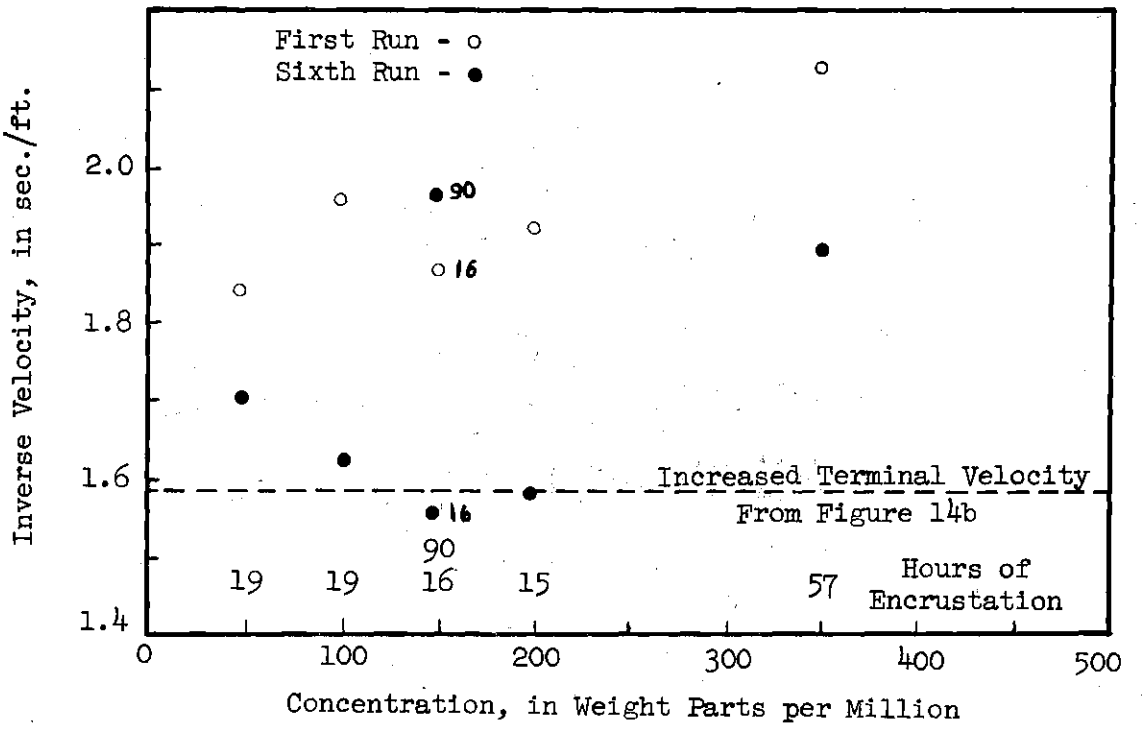


Figure 15b. Glass Spheres

Figure 15. Terminal Velocities of Polyhall Encrusted Spheres in Pure Water

and 70 hour intervals, and (c) dropped once into tubes of distilled and deionized water. The inverse terminal velocities are plotted in Figure 16 against drying time for the various concentrations. Although no curve can be drawn through these points, the initial drag showed a tendency to increase as the drying time increased to 50 hours. No relationship between drag and the concentration of the original polymer solution was discernible.

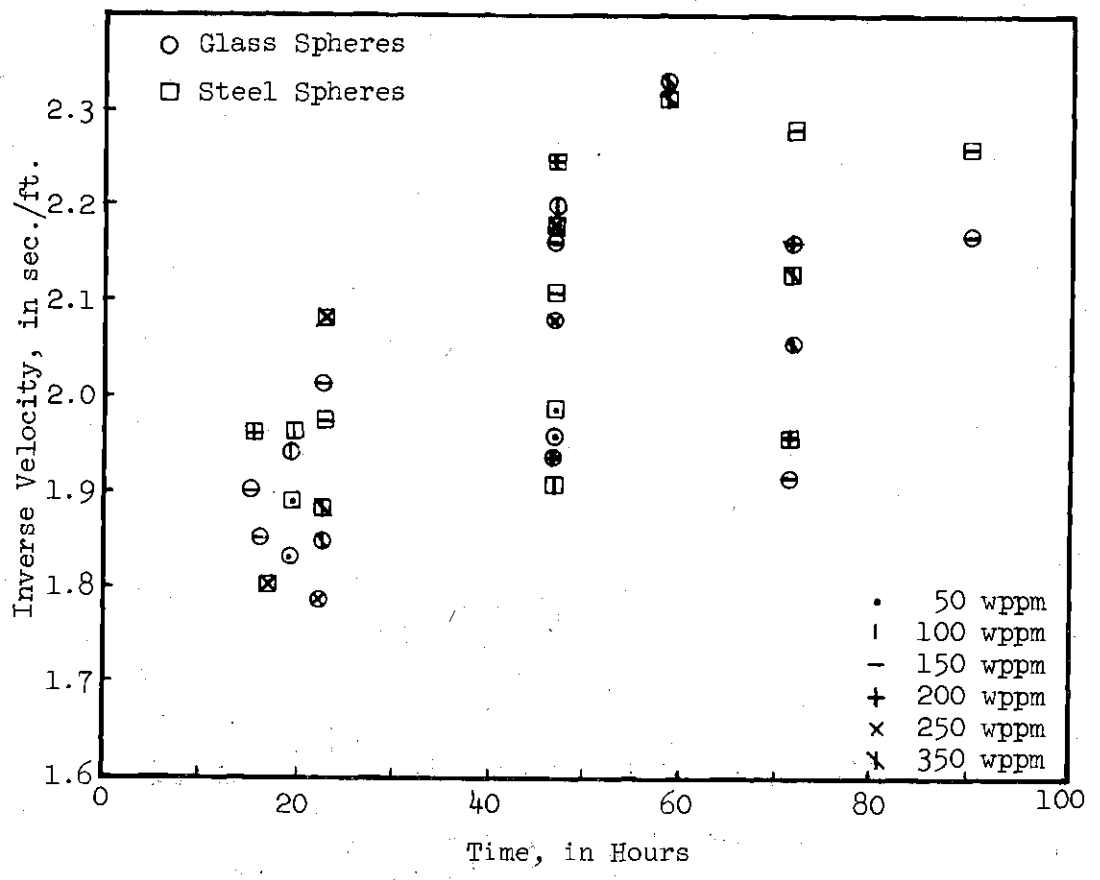


Figure 16. Time of Encrustation of Polymer Coated Spheres vs. Velocity and Concentration

The increase in drag recorded was apparently due to some effect of the encrusted polymer on the flow in the boundary layer. One other possibility is that the polymer macromolecules adsorbed on the surface

of the sphere protruded from the sphere at some angle to the sphere's surface. These protrusions, each approximately 5000 Anstrom units long, might have the gross effect of making the sphere hydraulically rough, thus causing an increase in the drag. The Reynolds number at which this result was recorded was approximately 3.4×10^3.

CHAPTER VII

CONCLUSIONS

The following conclusions are based on terminal velocity measurements taken for 5/8 inch diameter spheres falling in aqueous Polyhall 295 solutions at a Reynolds number of about 2.5×10^3.

1. The maximum drag reduction obtained for the falling spheres occurred at the same polymer concentration as the maximum drag reduction for the same polymer flowing in pipes, that is, $\sim$ 25 wppm (See Figures 1 and 11). It appears therefore that the drag reduction in both cases may be due to the same mechanisms.
2. The linear polymer Polyhall 295 is capable of producing drag reductions at Reynolds numbers below those of random-coiling polymers, for very low polymer concentrations.
3. The maximum drag reduction was 36 per cent for a polymer concentration of 25 wppm at a Reynolds number of 3760.

Other results for aqueous Polyhall 295 solutions determined from these experiments are:

1. As dilute aqueous Polyhall solutions are subjected to shear in a single direction, the polymer macromolecules in the solution align in the direction of the shear.
2. Aqueous Polyhall 295 solutions degrade with both mechanical shear and time.

CHAPTER VIII

RECOMMENDATIONS

1. Experiments should be performed with other linear polymers, to determine the minimum Reynolds number for which drag reduction can be expected by polymer addition.

2. A study should be performed to determine what effects Polyhall 295 solutions would have on the drag at $Re > 10^4$.

3. Further studies should be performed to determine if Polyhall 295 solutions exhibit a dual terminal velocity, as was noted by Sanders (11) for aqueous poly(ethylene oxide) solutions.

APPENDIX

SAMPLE CALCULATIONS

Viscosity Determination

In the Hoeppler Viscosimeter, the viscosity is linearly proportional to the time of fall of the ball in the tube. This relationship is given as:

$$C_\eta = T \cdot (S_b - S_f) \cdot B \tag{6}$$

in which C_η = Absolute viscosity in centipoises

T = Time interval of the falling ball

S_b = Specific gravity of the ball

S_f = Specific gravity of the fluid at the measuring temperature

B = Ball constant

In this experiment the ball used had a specific gravity of 2.2267 and a ball constant, B, of 0.07406. This experiment was performed at a constant temperature of 75°F. Thus

$$S_b - S_f = 2.2267 - 1.000 = 1.2267 \tag{7}$$

$$(S_b - S_f) \cdot B = 1.2267 \cdot 0.07406 = 0.09085 \tag{8}$$

Thus

$$C_\eta = 0.09085 \cdot T \tag{9}$$

To convert viscosity in centipoises to kinematic viscosity in ft.^{2}/sec. it is necessary to multiply the viscosity in centipoises by the proper conversion factors and then to divide this number by the density of the fluid. Using a value of 1.935 lb. sec.^{2}/ft.^{4} for the density yields:

$$1 \text{ centipoise} = 1.081 \times 10^{-5} \text{ ft.}^2/\text{sec.} \tag{10}$$

Thus the kinematic viscosity, ν, is given by:

$$\nu = 0.09085 \cdot T \cdot 1.081 \times 10^{-5} \tag{11}$$

$$\nu = 9.75 \times 10^{-7} \cdot T \tag{12}$$

Determination of C_D

The coefficient of drag has been shown to be equal to:

$$C_D = \frac{F_D}{\frac{1}{2} A \rho V^2} \tag{4}$$

For the apparatus of this experiment the drag force, F_D, can be given as:

$$F_D = F_W - (F_B + F_{cw} + F_f) \tag{13}$$

in which:

F_D = Drag Force

F_B = Buoyant Force

F_{cw} = Weight of Counterweight System

F_W = Weight of Sphere

F_f = Pulley Friction Force.

Thus, F_D can be determined by simple calculations of parameters of the system.

For the glass sphere, for example, substitution of values for and A yields:

$$C_D = \frac{F_D}{\frac{1}{2}\left(1.93 \frac{\text{lb. sec}^2}{\text{ft.}^4}\right) V^2 \left[\frac{\text{ft.}^2}{\text{sec}^2}\right] \left(2.0755 \text{ cm}^2\right)} \times \left(\frac{30.48 \text{ cm}}{\text{ft.}}\right)^2 \cdot \left(\frac{1 \text{ lb.}}{454 \text{ gm}}\right) \tag{14}$$

which yields:

$$C_D = \frac{F_D}{0.9823 \, V^2} \quad \text{where V is in ft/sec.} \tag{15}$$

LITERATURE CITED

1. Jackson, J.B., "Report on Experiments Involving Long-Chain Polymer Solutions," Department of Civil Engineering, Georgia Institute of Technology, (1967), (unpubl.).

2. White, A., "Turbulence and Drag Reduction with Polymer Additives," Hendon College of Technology, Research Bulletin, No. 4, 75, (1967).

3. Elata, C., Lehrer, J., and Kahanovitz, A., "Turbulent Shear Flow of Polymer Solutions," Israel Journal of Technology, 4, 87, (1966).

4. Ernst, W.D., "Investigation of the Turbulent Shear Flow of Dilute Aqueous CMC Solutions," American Institute of Chemical Engineers Journal, 12, No. 3, 581, (1966).

5. Hoyt, J.W. and Fabula, A.G., "The Effect of Additives on Fluid Friction," paper presented to Fifth Symposium on Naval Hydrodynamics, Bergen, Norway, 947, (1964).

6. Crawford, H.R. and Pruitt, G.T., "Rheology and Drag Reduction of Some Dilute Polymer Solutions," Symposium on Non-Newtonian Fluid Mechanics, Fifty-sixth Anniversary Meeting, A. I. Ch. E., Houston, Texas, (1963).

7. Ruszczycky, M.A., "Sphere Drop Tests in High Polymer Solutions," Nature, 206, 614, (1965).

8. Lang, T.G., "Drag of Blunt Bodies in Polymer Solutions," American Society of Mechanical Engineers, Publication No. 66-WA/FE-33, (1966).

9. White, D.A., "Drag Coefficients for Spheres in High Reynolds Number Flow of Dilute Solutions of High Polymers," Nature, 212, 277, (1966).

10. White, A., "Effect of Polymer Additives on Boundary Layer Separation and Drag of Submerged Bodies," Nature, 211, 1390, (1966).

11. Sanders, J.V., "Drag Coefficients of Spheres in Poly(ethylene oxide) Solutions," International Shipbuilding Progress, 14, No. 152, (1967).

12. Gadd, G.E., "Effects of Long-Chain Molecule Additives in Water on Vortex Streets," Nature, 211, 169, (1966).

13. Gadd, G.E., "Turbulence Damping and Drag Reduction Produced by Certain Additives in Water," Nature, 206, 463, (1965).

14. Vogel, V.M. and Patterson, A.M., "An Experimental Investigation of the Effect of Additives Injected Into the Boundary Layer of an Underwater Body," paper presented to Fifth Symp. Nav. Hydr., Bergen, Norway, 975, (1964).

15. McNown, J.S. and Nowlin, J.T., "Drag of Spheres Within Cylindrical Boundaries," Proceedings First U.S. National Congress of Applied Mechanics, Chicago, 801, (1951).

16. McNown, J.S., Lee, H.M., McPherson, M.B., and Engez, S., "Influence of Boundary Proximity on the Drag of Spheres," Proceedings VII Intl. Cong. Appl. Mech., London, 17, (1948).

17. Schlichting, H., Boundary Layer Theory, McGraw Hill Book Company, Inc., New York, Fourth Edition, (1960).

18. Alter, E.N., Laboratory Director, and Seaman, J.K., Chief Chemist, Natural Gum A&D Laboratory, Stein, Hall & Co., Inc., (personal communication).

19. Sanders, J.V., Professor - Naval Postgraduate School, Monterey, California, (personal communication).

Other References

Davies, G.A. and Ponter, A.B., "Turbulent Flow Properties of Dilute Polymer Solutions," Nature, 212, 66, (1966).

Elperin, I.T. and Smolski, B.M., "Effect of Surface Phenomena on Transfer Processes," Akademiya Navuk Belaruskey S.S.R., Vestsi, No. 2, 39, (1965).

Emerson, A., "Model Experiments Using Dilute Polymer Solutions," Transactions of the North-East Coast Institution of Engineers and Shipbuilders, 81, 201, (1965).

Fabula, A.G., "The Toms Phenomenon in the Turbulent Flow of Very Dilute Polymer Solutions," Proceedings of the Fourth International Congress on Rheology, Brown University, (1963).

Gadd, G.E., "Reduction of Turbulent Friction in Liquids by Dissolved Additives," Nature, 212, 874, (1966).

Gadd, G.E., "Differences in Normal Stress in Aqueous Solutions of Turbulent Drag Reducing Additives," Nature, 212, 1348, (1966).

Ghosh, S., "The Effect of Chemical Composition of ABS on Adsorption by Soils," M. S. Thesis, University of Illinois, (June, 1963).

Haberman, W. and Sayre, R.M., "Motion of Rigid and Fluid Spheres in Stationary and Moving Liquids Inside Cylindrical Tubes," Report 1143, David W. Taylor Model Basin, U.S. Navy Department, Washington, D.C., (1958).

Higgins, M.L., "The Viscosity of Dilute Solutions of Long-Chain Molecules: Part I," Journal of Physics and Chemistry, 42, 911, (1938).

Ibid., "Part II," 43, 439, (1939).

Kowalski, T., "Reduction of Frictional Drag by Non-Newtonian Additives," Naval Engineering Journal, 293, (1966).

Kramer, M.O., "The Dolphin's Secret," New Scientist, 7, 1118, (1960).

Lumley, J.L., "Turbulence in Non-Newtonian Fluids," Physics of Fluids, 7, No. 3, 335, (1964).

Ram, A., Finkelstein, E., and Elata, C., "Reduction of Friction in Oil Pipelines by Polymer Additives," I&EC Process Design and Development, 6, 309, (1967).

Ripken, J.F., and Pilch, M., "Studies of Reduction of Pipe Friction with the Non-Newtonian Additive CMC," St. Anthony Falls Hydraulic Laboratory, University of Minnesota, Technical Paper No. 42, Series B, (1963).

Rouse, P.E., Jr., "Viscoelastic Properties of Dilute Polymer Solutions of Coiling Polymers," Journal of Chemistry and Physics, 21, 1272, (1953).

Savins, J.G., "Drag Reduction Characteristics of Solutions of Macromolecules in Turbulent Pipe Flow," Society of Petroleum Engineers, Journal, 4, 203, (1964).

Shapiro, A.H., Shape and Flow - The Fluid Dynamics of Drag, Doubleday and Co., Inc., New York, (1961).

Tennekes, H., "Wall Region in Turbulent Shear Flow of Non-Newtonian Fluids," Physics of Fluids, 9, 872, (1966).

Toms, B.A., "Some Observations on the Flow of Linear Polymer Solutions through Straight Tubes at Large Reynolds Numbers," Proceedings, First International Congress on Rheology, Holland, II, 135, (1948).

Wells, C.S, Jr. and Spangler, J.G., "Injection of a Drag Reducing Fluid Into Turbulent Pipe Flow of a Newtonian Fluid," LTV Research Center, Dallas, Texas, Report No. 0-71000/7R-4, (1967).

White, A., "Turbulent Drag Reduction with Polymer Additives," Journal of Mechanical Engineering Science, 8, 452, (1966).